M421
Exam 2
Monday Nov. 13

NAME: ______________________________

1	2	3	4	TOTAL

The value of each question is indicated next to the question.
No calculators, books, or notes are permitted.
Except where indicated otherwise, you may use any Theorem we have proved in this class, or any theorem from differential calculus (Mean value theorem, Rolle's theorem, L'Hopital's rule, etc).
You may not quote results from HW – i.e. you may not say " As proved in my HW3 we know that ..."
Show your work to receive partial credit.
If you need extra space, write on the back of the page.
No hitting, biting, or fighting.

1) (10pts) Let $f : \mathbf{R}^n \mapsto \mathbf{R}^m$.

(a) State the definition for f to be differentiable at $\vec{x} = \vec{a}$.

(b) State what you must show to prove that f is **not continuous** at $\vec{x} = \vec{a}$.

2) (10pts) (a) State the definition for a sequence $\{\vec{x}_k\}_{k=1}^{\infty} \subset \mathbf{R}^n$ to be Cauchy.

(b) From the definition of convergence prove that $\lim_{k\to\infty} \vec{x}_k = \vec{a}$ implies $\{\vec{x}_k\}_{k=1}^{\infty}$ is Cauchy.

3) (15pts) Let $f : [a,b] \times [c,d] \subset \mathbf{R}^2 \mapsto \mathbf{R}$ be continuous. Show that

$$F(y) = \int_a^y f(x,y)dx$$

is uniformly continuous on $[c,d]$.

4) (15pts) Consider the function $f : \mathbf{R}^2 \mapsto \mathbf{R}$ given by

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2} + y & \vec{x} \neq 0, \\ 0 & \vec{x} = 0 \end{cases}$$

(a) Calculate $D_{\vec{v}} f(0)$ for each $\vec{v} \in S^1$.

(b) Show that $\frac{\partial f}{\partial x}(\vec{x} = 0)$ and $\frac{\partial f}{\partial y}(\vec{x} = 0)$ exist and evaluate them.

(c) Determine if f is or is not differentiable at $\vec{x} = 0$.