



M421 HW 4

Due Friday Oct. 26

From Wade

Section	Page Number	Problems
9.2	269-270	3(ab)
9.3	275-277	4, 5, 8
9.4	279	2, 3, 6

Non-book Exercises

1) For two sets $A, B \subset \mathbf{R}^n$ define

$$\text{dist}(A, B) = \inf_{\vec{x} \in A, \vec{y} \in B} \|\vec{x} - \vec{y}\|.$$

(a) Show that if $E \subset \mathbf{R}^n$ is closed and $K \subset \mathbf{R}^n$ is compact then

$$E \cap K = \emptyset, \quad \iff \quad \text{dist}(E, K) > 0.$$

(b) Find two closed sets E_1 and E_2 in \mathbf{R}^2 such that $\text{dist}(E_1, E_2) = 0$ but $E_1 \cap E_2 = \emptyset$.

2) Suppose that $E \subset \mathbf{R}^n$ is connected and $E \subset A \subset \overline{E}$. Prove that A is connected.

3) Let f be defined on \mathbf{R}^2 by

$$f(x, y) = \frac{x|y|^\alpha}{x^2 + y^2}.$$

For which $\alpha > 0$ is it true that

$$\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0.$$