

Note that $r=\rho+\epsilon$ so that $r=\epsilon$ is now $\rho=0$. f is the Exact eigenfunction for the punctured disk, g is the composite expansion that we derived in class, and h is the Exact eigenfunction for the unit disk (no hole).

```
> f:=1/(rho+epsilon)*(sin(Pi*(rho+epsilon))-tan(Pi*epsilon)*cos(Pi*(rho+epsilon)));
```

$$f := \frac{\sin(\pi(\rho + \epsilon)) - \tan(\pi\epsilon) \cos(\pi(\rho + \epsilon))}{\rho + \epsilon}$$

```
> g:=sin(Pi*(rho+epsilon))/(rho+epsilon) - epsilon*Pi/(rho+epsilon);
```

$$g := \frac{\sin(\pi(\rho + \epsilon))}{\rho + \epsilon} - \frac{\epsilon \pi}{\rho + \epsilon}$$

```
> h:=sin(Pi*(rho+epsilon))/(rho+epsilon);
```

$$h := \frac{\sin(\pi(\rho + \epsilon))}{\rho + \epsilon}$$

```
> plot(subs(epsilon=exp(-6),{f,g,h}),rho=0..1,numpoints=1000,thickness=2,color=black);
```



