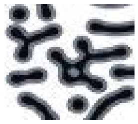

Math 942

HW1 – Due Monday Sept. 17

From Holmes

Section	Page Number	Problems
1.5	23-25	1(aejp), 3, 5, 7
1.6	31-34	1(acf), 4, 5, 7

Non-book Exercises

1) Consider the perturbed polynomial

$$P_\epsilon(x) = P_0(x) + \epsilon g(x),$$

where P_0 has a zero of order k at $x = x_0$ and g has a zero of order $j < k$ at x_0. Show that P_ϵ has a zero of order j at x_0 and find the correction $x_\epsilon = x_0 + \epsilon^q x_1$ for the zeros which move away from x_0, (that is find q and an expression for x_1).

2) Consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix},$$

and

$$B = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

The matrix A has a two-dimensional eigenspace associated to the eigenvalue $\mu = 1$. Find the leading order correction (i.e. the two term expansion) for the two eigenvalues (and the associated eigenvectors) of the eigenspaces of $C = A + \epsilon B$, which perturb from $\mu = 1$ eigenspace of A.

3) Consider the eigenvalue problem

$$-u'' + \epsilon F(x)u = \lambda u \quad 0 < x < 1,$$

$$u(0) = u(1), \qquad u'(0) = u'(1),$$

where F is a smooth, known function. Find a formula for the correction to the $n'th$ eigenvalue λ_n of the unperturbed problem. Hint: Show the problem is self-adjoint. With the periodic boundary conditions each unperturbed eigenvalue has two eigenfunctions, so λ_n may split upon perturbation!