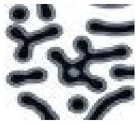

Math 942

HW3 – Due Monday Nov. 16

From Holmes

Section	Page Number	Problems
2.6	94-98	1, 8, 11
3.2	113-115	1(b), 7
3.3	122	6
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Non-book Exercises

1) Consider the generalization of the pipe-flow temperature problem from class:

$$\epsilon\left(U_{rr} + \tfrac{1}{r}U_r + U_{xx}\right) = -V(r)U_x, \qquad 0 \le r \le 1, -\infty < x < \infty,$$
$$U(x,1) = \begin{cases} 0 & x > 0, \\ 1 & x < 0 \end{cases}$$
$$U \to 0 \text{ as } x \to \infty.$$

The function V satisfies $V(r) > 0$ for $0 \le r < 1, V(1) = 0$, and $V(r) = \gamma(r-1)^{2\alpha+1} + \dots$ as $r \to 1$, for α some non-neg. integer. Find a leading order expansion for the temperature U in the pipe.

2) Consider the two-dimensional problem

$$\epsilon\Delta u = u_x, \qquad 0 < r < 1,$$
$$u(1,\theta) = \begin{cases} 1 & 0 < \theta < \pi/2 \\ 0 & \pi/2 < \theta < 3\pi/2 \\ -1 & 3\pi/2 < \theta < 2\pi \end{cases}$$

where $\theta = 0$ is the positive x axis. Find a leading order expansion which includes all boundary layers, (you may neglect the regions of intersection of two layers).

3) Consider the eigenvalue problem,

$$\Delta u + \lambda u = 0, \qquad \text{in } D_\epsilon$$
$$u = 0 \qquad \text{on } \partial D_\epsilon,$$

where $D_\epsilon = \{x = (x_1, x_2, x_3)\Big|\epsilon < |x| < 1\}$. Find an expression for the smallest eigenvalue $\lambda_0(\epsilon)$. To which eigenvalue problem does $\epsilon = 0$ correspond (Laplacian in a sphere or a punctured sphere)? Hint: think innies and outies.